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RADIAL MASS TRANSFER EFFECTS IN A POROUS WALL TUBULAR REACTOR

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Abstract—A porous wall tubular reactor with selective suction or injection of the reactant and/or the product and/or the inert offers many advantages over a conventional solid wall tubular reactor. However, the usage of this type of reactor requires a thorough knowledge of the effects of radial mass transfer on the performance of the reactor. The present paper analyzes these effects under plug as well as laminar flow conditions and for an isothermal, first order irreversible reaction process. Both bulk phase and surface catalyzed reaction processes are considered. The results of the analysis are presented and interpreted in terms of the quantity of feed reactant depleted by the reaction within the reactor, under a variety of reaction, flow and radial mass transfer conditions.

NOMENCLATURE

A,	reactant;
B,	product;
B_n,	series constants defined by equation (24);
c,	reactant concentration [lbm/ft^3];
C,	dimensionless reactant concentration, c/c_0;
D_A,	molecular diffusion coefficient of reactant A in the reacting mixture [ft^2/h];
f_A,	a quantity defined by equations (7) and (25);
$g(Y)$,	a quantity defined by equation (19);
G_n,	radial eigenfunctions;
F_A,	flux of reactant across the tube wall [lbm/ft^2 h];
$\bar{F}_A$,	dimensionless flux of reactant across the tube wall $(F_A/v_w c_{A0})$;
k,	intrinsic reaction rate constant, units for bulk phase reaction [1/h], units for surface catalyzed reaction [ft/h];
K_1,	dimensionless kinetic constant as defined by equations (3) and (9);
L,	dimensionless axial distance (x/R);
P,	permeation Reynolds number (Rv_w/ν);
r,	radial distance from the center of the tube [ft];
R,	tube radius [ft];
S,	rejection coefficient $(1 - F_A/v_w c_{Aw})$;
u,	axial velocity [ft/h];
$\bar{u}(0)$,	average axial velocity at the tube inlet [ft/h];
U,	dimensionless axial velocity $(u/\bar{u}(0))$;
v,	radial velocity [ft/h];
v_w,	suction or injection velocity at the tube wall [ft/h];
V,	dimensionless radial velocity (v/v_w);
x,	axial distance from the tube inlet [ft];
X,	a quantity defined by equation (3);
Y,	dimensionless radial distance from tube center (r/R).

Greek symbols

α,	normalized diffusion coefficient $(D_A/v_w R)$;
β_n,	eigenvalues;
γ,	a quantity defined by equation (9);
δ,	$v_w/\bar{u}(0)$;
ν,	kinematic viscosity [ft^2/h];
ϕ,	weighting function defined by equation (23);

Φ, a quantity defined by equation (3).

Subscripts

0, condition at the tube inlet;
w, condition at the tube wall;
A, quantity referring to reactant.

INTRODUCTION

THE BASIS for designing a chemical reactor is to obtain a suitable equipment in which a reaction can be carried out under the controlled and far away from equilibrium conditions for the reacting mixture. Over the past years, several types of reactors have arisen to meet the needs for carrying out a wide variety of reactions in economic and convenient fashions. One of the reactors of current research interest is a porous wall reactor. This novel type of reactor consists of a tube whose walls are made of a finely porous material through which suction or injection of inert material or reactant can be carried out. This reactor, thus, allows one to achieve axially uniform concentration and temperature distributions which are desirable for improving the performance of a tubular reactor.

So far, very little has been published on the theoretical as well as experimental aspects of a porous wall reactor. Weger and Effron [1] examined the cooling of aerodynamic surfaces by endothermic reactions on a porous tube impregnated with catalyst. Hartnett and Eckert [2] studied the theoretical considerations involved in the combustion of a fluid coolant after passage through a porous wall, and Satterfield *et al.* [3] investigated the decomposition of hydrogen peroxide vapor in a surface catalyzed porous wall tubular reactor.

A porous wall reactor offers many advantages over a solid wall reactor. If it is run with suction at the wall, the pressure drop across the reactor length, would be lower than that for solid wall reactor for the same inlet flowrate [4, 5]. If the reaction generates an excessive amount of heat, the injection or suction of a fluid can reduce the excessive heating of the reactor. The driving force for the reaction or the reactant concentration can be maintained at a constant level along the entire length of a tubular reactor with the help of injection or suction at the wall. A porous wall reactor with suction at the wall can be used to separate product or inert from the reacting mixture during the reaction process.

The control of axial concentration distribution in a porous wall reactor can be achieved by selectively injecting or removing reactant or product (or inert) at the wall. This selective rejection or injection would, in general, cause radial gradients in concentrations. The purpose of the present paper is to analyze the performance of a porous wall reactor under a wide variety of radial concentration gradients of the reactant and thereby to evaluate the range of operating conditions under which this type of reactor will truly give a better performance over a solid wall reactor. The analysis is carried out for the first order isothermal reaction process in plug and laminar flow conditions. Both bulk phase or homogeneous and the surface catalyzed reaction processes are considered.

THEORETICAL

Plug flow

The ideal well mixed plug flow or the condition of zero radial concentration and velocity gradients represent the most simple and the extreme case of the present analysis. Since from the practical stand point, the knowledge of the performance of a porous wall reactor under this ideal condition is sometimes very desirable, an analysis of a porous wall reactor for this case is briefly examined here.

For a porous wall reactor such as one illustrated by Fig. 1, the differential mass balance on the reactant A, when the reaction is first order and irreversible, can be written in the dimensionless form as:

$$\frac{\mathrm{d}\,(UC)}{\mathrm{d}X} + K_1 C + \bar{F}_A = 0. \qquad (1)$$

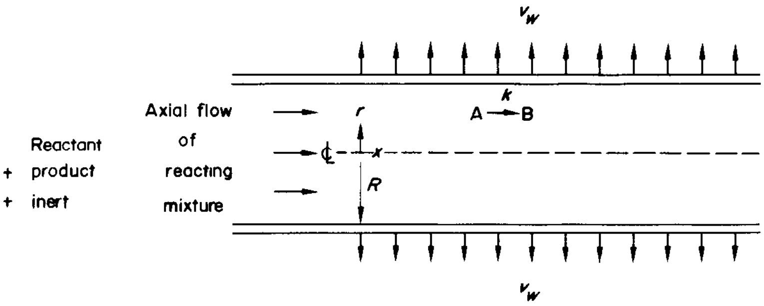

FIG. 1. Schematic of a porous wall tubular reactor.

Also, the overall mass balance will give

$$U = 1 - X\Phi \tag{2}$$

where,

$$X \equiv [2v_w/\bar{u}(0)] \quad (x/R)$$

$$U \equiv u/\bar{u}(0)$$

$$C \equiv c_A/c_{A0}$$

$$\bar{F}_A \equiv F_A/v_w\, c_{A0} \tag{3}$$

$$\Phi \equiv +1 \text{ for suction}$$

$$\Phi \equiv -1 \text{ for injection}$$

$$K_1 \equiv [kR/v_w] \text{ for bulk fluid phase or homogeneous reaction}$$

$$K_1 \equiv k/v_w \text{ for surface catalyzed reaction.}$$

In deriving equations (1) and (2) it is assumed that the rate expression for the disappearance of reactant A for both homogeneous and surface catalyzed reactions is given by the form

$$\text{rate of disappearance of reactant } A = -k\, c_A. \tag{4}$$

It should be noted that the units for k are different for the cases of homogeneous and surface catalyzed reactions. In equation (3) F_A is the permeation flux of reactant A at the reactor wall.

Equations (1) and (2) also assume that the volume change upon mixing and reaction is negligible and that natural convection and axial diffusion effects are negligible. The total permeation velocity at the tube wall is assumed to be constant and uniform.

If permeation flux F_A, is assumed to be constant over the entire length of the reactor, the solution of equations (1) and (2) with the condition at the reactor inlet that at $X = 0$, $C = 1$, would give

$$C = \frac{\bar{F}_A}{(1 - K_1\Phi)} + \left(1 + \frac{\bar{F}_A}{K_1\Phi - 1}\right)^{(1 - X\Phi) - (1 - K_1\Phi)} \tag{5}$$

It should be noted that the quantity X as defined in equation (3) represents the ratio of the total permeation through the wall to the inlet flowrate. For the case of suction this quantity can never exceed 1. Hence for the case of suction equation (5) is valid only for the values of X between 0 and 1. If a quantity f_A is defined as,

$$f_A = \frac{\text{(Inlet flow of reactant} \pm \text{flow of reactant across the tube wall} - \text{flow of reactant at a given axial distance)}}{\text{Inlet flow of reactant}} \tag{6}$$

where second term in the numerator of above equation has the plus sign for the case of injection and minus sign for the case of suction, then this quantity gives the magnitude of the fraction of the inlet reactant flowrate that is depleted by the reaction in a reactor of given length. It should be noted that for the case of injection this quantity can be greater than 1. In the present study f_A is assumed to be an index for the reactor performance and the effects of reaction, flow and the radial mass transfer conditions on its value are examined. An expression for f_A in terms of reactor variables defined by equation (3) can be obtained from the overall mass balance on the reactant. The final result is

$$f_A = 1 - \bar{F}_A \Phi X - (1 - X\Phi)\left[\frac{\bar{F}_A}{(1 - K_1\Phi)} + \left(1 + \frac{\bar{F}_A}{K_1\Phi - 1}\right)(1 - X\Phi)^{-(1-K_1\Phi)}\right]. \quad (7)$$

Laminar flow

One would expect from the physical considerations of a porous wall reactor that the laminar flow condition will give the prominent radial gradients of concentrations and velocity within the reactor. Hence, the analysis of a porous wall reactor is more interesting for this flow condition than for the ideal plug flow case. This analysis is carried out here for a tubular geometry such as one illustrated by Fig. 1. The final results of this analysis should, however, be qualitatively applicable to other similar geometries i.e. a rectangular duct etc. The following assumptions are made:

1. fluid is incompressible and that fluid properties such as density, viscosity and diffusion coefficient are constant
2. natural convection and axial diffusion effects are negligible
3. volume change upon mixing and reaction is negligible
4. permeation velocity at the tube wall is constant and uniform
5. laminar flow is fully developed at the channel inlet

With the above assumptions, the mass balance on the reactant A in the dimensionless form can be written as,

$$\frac{1}{\delta}\frac{\partial}{\partial L}(UC) + \frac{1}{Y}\frac{\partial}{\partial Y}\left[Y\left(VC - \alpha\frac{\partial C}{\partial Y}\right)\right] + K_1 C\gamma = 0 \quad (8)$$

where

$$\begin{aligned} L &\equiv x/R \\ Y &\equiv r/R \\ C(L, Y) &\equiv c_A(x, r)/c_{A0}(x, r) \\ U(L, Y) &\equiv u(x, r)/\bar{u}(0) \\ \alpha &\equiv D_A/v_w R \qquad (9) \\ \delta &\equiv v_w/\bar{u}(0) \\ V(Y) &\doteq v/v_w \end{aligned}$$

$\gamma \equiv 1$ bulk fluid phase or homogeneous reaction

$\gamma \equiv 0$ surface catalyzed reaction

$K_1 \equiv kR/v_w$

where k is defined by equation (4).

It should be noted that the ratio of the wall permeation to the inlet flowrate, X, would be numerically equal to $2\delta L$. In order to solve equation (8), a velocity field must be known. For the assumptions made in the present study, Yuan and Finkelstein [5] have shown that for a permeation Reynolds number, P (which is numerically equal to $v_w R/\nu$), less than 1, the velocity field in the dimensionless form can be expressed as,

$$V(Y) = 2Y\Phi[1 - (Y^2/2) - (P\Phi/72)(-4 + 9Y^2 - 6Y^4 + Y^6) + (P^2/10800)(166 - 380Y^2 + 275Y^4 - 75Y^6 + 15Y^8 - Y^{10})] \quad (10)$$

and

$$U(L, Y) = 2[1 - 2\delta L\Phi][1 - Y^2 - (P\Phi/36)(-2 + 9Y^2 - 9Y^4 + 2Y^6) + (P^2/10800)(166 - 760Y^2 + 825Y^4 - 300Y^6 + 75Y^8 - 6Y^{10})]. \quad (11)$$

Since the value of permeation Reynolds number less than 1 would be of the most practical interest, in the present analysis the velocity field given by equations (10) and (11) is used for the solution of equation (8).

BOUNDARY CONDITIONS

A. *Suction*

The initial and boundary conditions for the case of suction can be written in the manner similar to one demonstrated by Sherwood *et al.* [6] in the study of concentration polarization in reverse osmosis. Thus,

$$C(0, Y) = 1{\cdot}0 \tag{12}$$

$$(\partial C/\partial Y)_{Y=0} = 0{\cdot}0 \tag{13}$$

and

$$(S - K_1\gamma)\, C(L, 1) = \alpha(\partial C/\partial Y)_{Y=1} \tag{14}$$

where

$$K_1 \equiv k/v_w.$$

The quantity γ in equation (14) has the same meaning as one defined in equation (9). The quantity S in equation (14) is equal to 1 minus the ratio of the reactant flux through the wall to the product of the suction velocity and the reactant concentration at the tube wall. This quantity is assumed to be constant over the entire length of the reactor [7].

B. *Injection*

The initial condition and the boundary condition at the tube center for this case would be the same as ones given by equations (12) and (13). However, the boundary condition at the tube wall would be

$$\bar{F}_A + (1 - K_1\gamma)\, C = \alpha(\partial C/\partial Y)_{Y=1}. \tag{15}$$

The quantity γ in above equation has once again the same meaning as one defined in equation (9). $\bar{F}_A$ is the dimensionless quantity of reactant A injected into the system. It has the same meaning as one defined in equation (3). Since in the case of injection, the radial concentration gradient is most prominent when only pure inert is injected, $\bar{F}_A$ in the present analysis is assumed to be equal to zero.

The solution of the system of equations (8)–(15) was obtained in the present study by the method of separation of variables. Thus, assuming

$$C(L, Y) = f(L)\, G(Y) \tag{16}$$

it could be shown that

$$C(L, Y) = \sum_{n=0}^{\infty} B_n(1 - 2\delta\, L\Phi)^{\beta_n/4 - 1}\, G_n(Y) \tag{17}$$

where β_n and $G_n(Y)$ are the eigenvalues and the solutions of

$$\frac{\mathrm{d}}{\mathrm{d}Y}\left[Y\left(VG_n - \alpha\frac{\mathrm{d}G_n}{\mathrm{d}Y}\right)\right] = g(Y)\, Y\beta_n G_n(Y) - K_1\gamma\, YG_n(Y) \tag{18}$$

where

$$\begin{aligned} g(Y) = {} & [1 - Y^2 - (P\Phi/36) \\ & \times (-2 + 9Y^2 - 9Y^4 + 2Y^6) \\ & + (P^2/10\,800)(166 - 760Y^2 + 825Y^4 \\ & - 300Y^6 + 75Y^8 - 6Y^{10})]. \end{aligned} \tag{19}$$

The boundary conditions are

$$\frac{\mathrm{d}G_n}{\mathrm{d}Y} = 0 \qquad \text{at } Y = 0 \tag{20}$$

and at $Y = 1$

$$\alpha\frac{\mathrm{d}G_n}{\mathrm{d}Y} = (S - K_1\gamma)\, G_n \text{ for suction} \tag{21}$$

$$\alpha\frac{\mathrm{d}G_n}{\mathrm{d}Y} = (1 - K_1\gamma)\, G_n \text{ for injection of inert.} \tag{22}$$

In equations (18), (21) and (22)

$\gamma = 0$ for bulk phase reaction

$\gamma = 1$ for surface catalyzed reaction

The series constants B_n are evaluated from the orthogonality of the radial eigenfunctions. The weighting function is [8].

$$\phi = \frac{g(Y)\,Y}{\alpha}\exp\left[-\frac{1}{\alpha}\int \Phi V(Y)\,\mathrm{d}Y\right] \qquad (23)$$

and

$$B_n = \frac{\int_0^1 \phi G_n(Y)\,\mathrm{d}Y}{\int_0^1 \phi G_n^2(Y)\,\mathrm{d}Y}. \qquad (24)$$

Equation (18) was solved by the method of Frobenius [9]. The infinite series of equation (17) was expanded up to first 100 terms. The method of solution of equation (8) involved three phases. The first phase generated the eigenvalues. The second phase generated the radial eigenfunctions and the series constants. Finally, the third phase generated the concentration field. The calculations for all three phases were carried out on an IBM 360 digital computer. The eigenvalues and series constants for typical limiting conditions (when $k \rightarrow 0$) agreed well with those reported by Fisher *et al.* [10]. The series given by equation (17) converged faster at far downstream as opposed to near the tube entrance. The concentration profiles were thus obtained for several values of α, K_1, X and S. Once the concentration profiles were known, the values for the index of the reactor performance, f_A, were calculated from the following expression:

$$f_A = 1 - 2(1 - X)\int_0^1 UCY\,\mathrm{d}Y$$
$$- \int_0^X (VC - \alpha\,\partial C/\partial Y - K_1\gamma C)_{Y=1}\,\mathrm{d}X \qquad (25)$$

where γ has the same meaning as the one described earlier. The integrations in equation (25) were evaluated by the Simpson's rule of integration [8]. It should be noted that the third term on the right hand side of equation (25) will be equal to zero for the case of injection of inert. When the reactor wall is impervious ($v_w \rightarrow 0$), the mathematical equations for the laminar flow condition outlined here will reduce to the ones reported by Cleland and Wilhelm [11] and for this case the present analysis gave the results that agreed well with the ones reported by Cleland and Wilhelm [11].

RESULTS AND DISCUSSION

The purpose of the present analysis was to evaluate the performance of an isothermal porous wall reactor under a wide variety of transverse or radial mass transfer conditions and thereby to make a detail assessment of the various important features of a porous wall reactor.

The condition of well-mixed plug flow or that of no radial concentration and velocity gradients is an ideal case of the present study. Since this condition may be desirable in many practical instances, the results for f_A (which is considered in the present study as an index for the reactor performance) shown in equation (7) were numerically evaluated. In order that the performance of a porous wall reactor can be compared with that of a solid wall reactor under the similar reaction conditions, the values for f_A were evaluated as a function of the quantity K_1X. The product K_1X is numerically equal to the dimensionless kinetic constant, $(kx/\bar{u}(0)$, for a solid wall plug flow reactor [12].

Figure 2 shows the effect of the total amount of permeation through the reactor wall on the performance of a porous wall reactor. The results shown in this figure are for the case when there is no transfer of reactant across the reactor wall. Thus, for the case of suction, the permeation of only product and inert will increase the concentration of the reactant in the reactor and this will, of course, improve the performance of the reactor. Also, as shown quantitatively in Fig. 2, the improvement in the performance would be more for the larger amount of permeation. This type of effect of suction on the reactor performance for the first order as well as other types of reaction mechanism and for a thin channel geometry is qualitatively described recently by Shah *et al.* [13].

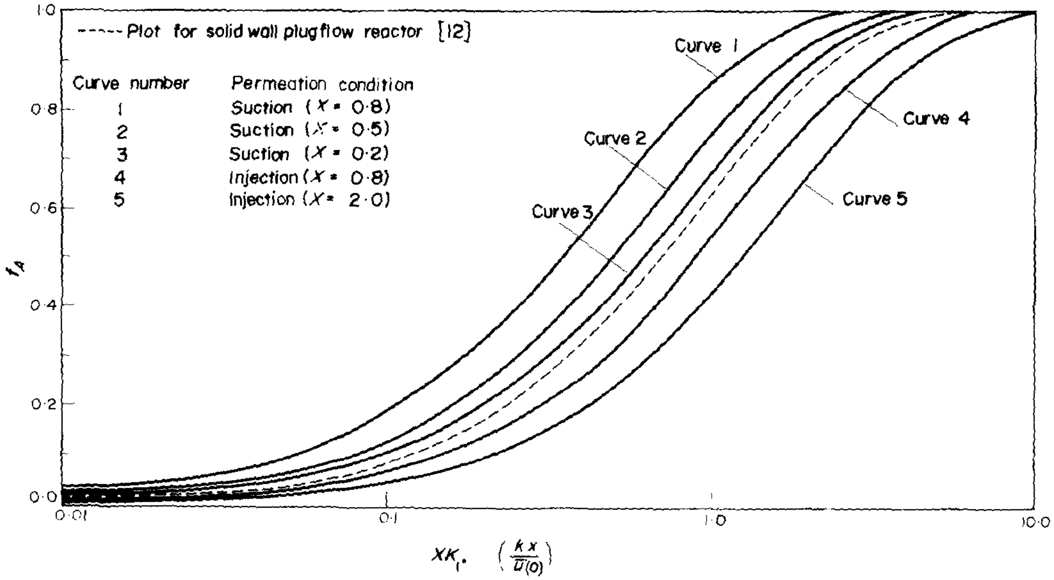

FIG. 2. Performance of a porous wall reactor under plug flow condition—no transfer of reactant across the reactor wall.

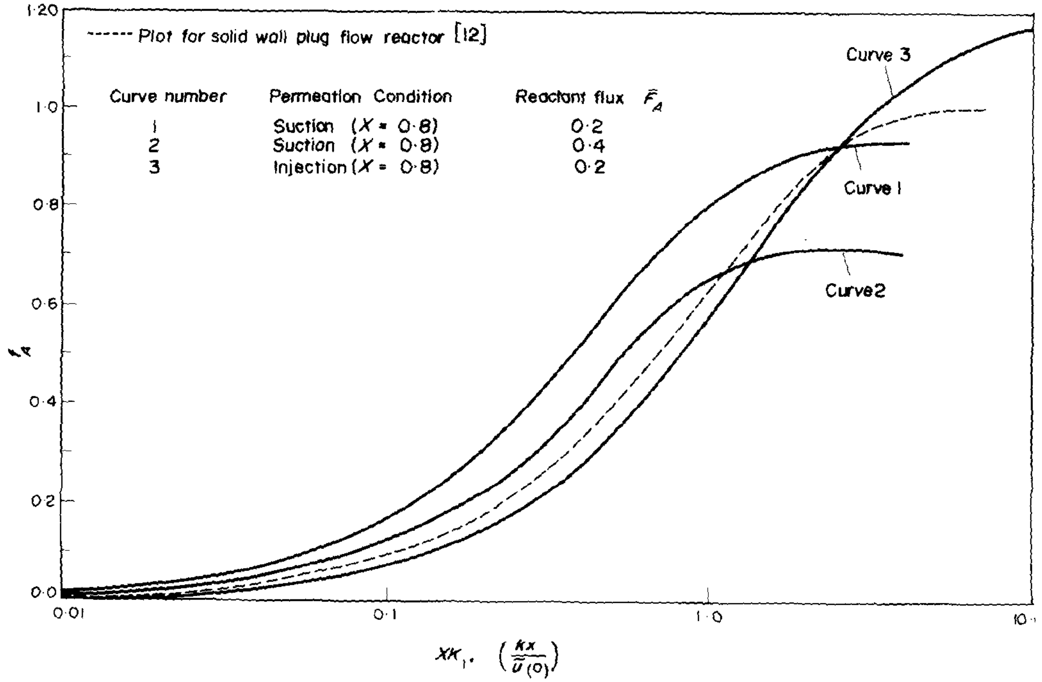

FIG. 3. Performance of a porous wall reactor under plug flow condition—effect of reactant transfer across the reactor wall.

Unlike suction, the injection of inert will degrade the performance of a plug flow reactor because in this case the reacting mixture is diluted. Also, higher the amount of injection, more degradation in the reactor performance would occur. This effect is quantitatively described in Fig. 2. It should be noted that the results shown in Fig. 2 can be applied to both bulk phase as well as surface catalyzed reaction processes if the proper definition for the dimensionless kinetic constant [as defined by equation (3)] is used. It is also interesting to note from Fig. 2 that if the kinetic constant for the reaction is very small or very large, the permeation of inert (or product) through the reactor wall does not significantly change the reactor performance.

From practical stand point, the transfer of reactant across the reactor wall along with that of inert and/or product is almost inevitable. In case of suction the leaking of reactant through tube wall will obviously hurt the reactor performance. On the other hand, in case of injection, permeation of the reactant through the tube wall will help in improving the performance of the reactor. The effects of transfer of reactant across the tube wall on the f_A vs. K_1X plots are described in Fig. 3. This figure describes the above mentioned effects quantitatively for a value of X equal to 0·8 and for the values of $\bar{F}_A$ equal to 0·2 and 0·4.

Laminar flow

In case of laminar flow, since the radial gradients in the velocity and the concentrations are finite and non-zero except at the tube center, the analysis of a porous wall reactor is more interesting and realistic. The magnitudes of the radial concentration gradients in this case will depend upon the value of the normalized diffusion coefficient, α. Hence, the performance of a porous wall reactor in laminar flow regime will depend upon the magnitude of α along with those of kinetic constant K_1, the total permeation X and the reactant permeation through the tube wall. The effects of these variables on the reactor performance are briefly evaluated below. The cases of suction and the injection as well as those of bulk fluid reaction and the surface catalyzed reaction are treated separately.

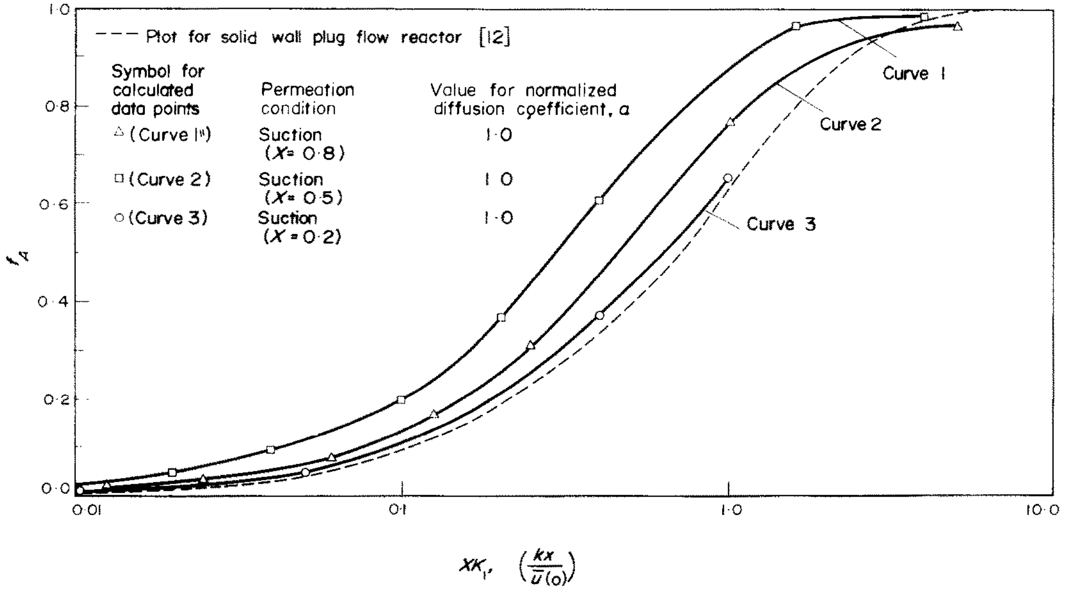

FIG. 4. Performance of a porous wall reactor in laminar flow condition—effect of amount of wall permeation of inert and/or product in case of bulk fluid phase reaction process.

Suction with bulk phase reaction

As noted earlier, in the analysis of a plug flow reactor that the porous wall characteristics in the case of suction will improve the performance of a reactor best when there is no permeation or leaking of the reactant through the wall. Hence the numerical results for the solution of equations (8)–(25) were first obtained for this condition.

Figure 4 shows the plots of f_A [as evaluated from equation (25)] vs. K_1X for three different values of X, a typical value of α equal to 1 and for the condition of no permeation of reactant through the tube wall. It should be noted that K_1 in this and all the rest of the similar plots concerning with bulk phase reaction process is one defined in equation (3) and not the one defined in equation (9). The difference in the kinetic constants defined in these two equations can be noted to be a factor of 2. The plots shown in Fig. 4 indicate that, in general, just like in the plug flow case, an increasing amount of the improvement in the reactor performance will be obtained with an increase in wall permeation. This is to be expected because even though in this case there exist the large radial concentration gradients (see Fig. 5), the cup-mixing concentrations are still higher than those for the solid wall reactor under the identical reaction conditions. Also for the same reaction conditions the cup-mixing concentration at a given axial distance in a porous wall reactor will be higher for the larger quantity of suction of inert through the wall.

The more interesting aspect of the laminar flow analysis is the effect of the magnitude of normalized diffusion coefficient, α, on the reactor performance. This effect for a value of X equal to 0·5 and for the condition of complete rejection of reactant at the tube wall is illustrated in Fig. 6. The dotted curve in this figure is once again a plot of f_A vs. dimensionless kinetic constant, $(kx/\bar{u}(0))$, for a solid wall plug flow reactor. A range of values of α from 0·067 to 3 is considered in this figure. A plot of f_A vs. K_1X for the case of well mixed plug flow ($\alpha = \infty$) in the porous reactor is also shown in this figure. These results indicate that as the valuc of α increases the plot of f_A vs. K_1X, as expected, becomes more and more like the one for α equal to ∞. As a matter of fact the plot of f_A vs. K_1X for the value of α equal to 3 was

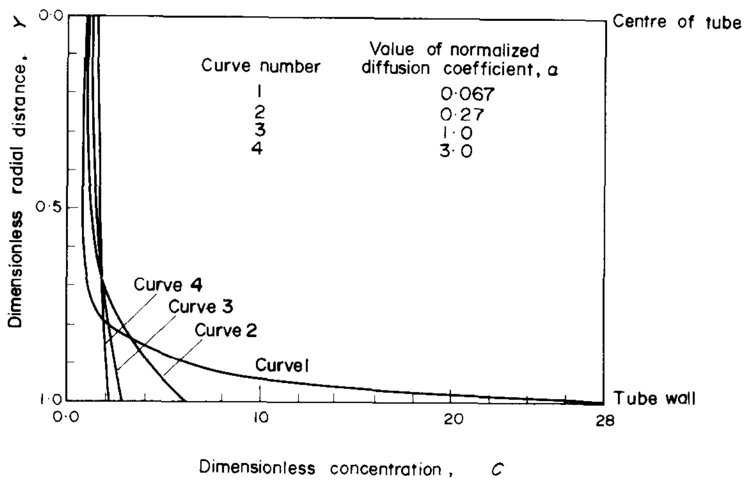

FIG. 5. Typical radial concentration profiles in a porous wall reactor with suction (X = 0·5, K_1 = 0·25 and S = 1·0)—bulk fluid phase reaction process in laminar flow condition.

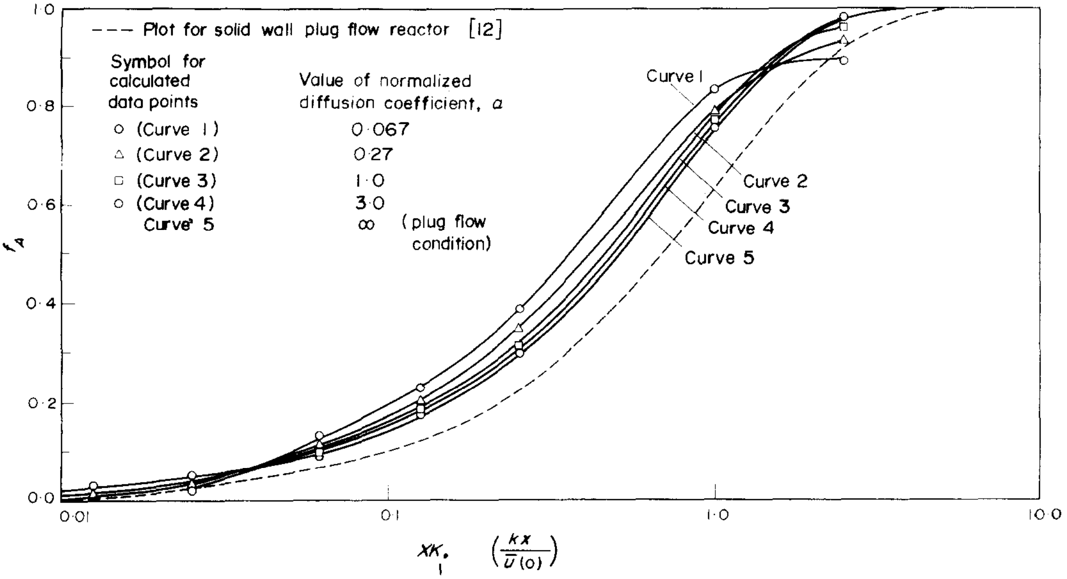

FIG. 6. Effect of normalized diffusion coefficient on the performance of a porous wall reactor in case of suction for a value of $X = 0{\cdot}5$ ($S = 1{\cdot}0$)—bulk fluid phase reaction process in laminar flow condition.

found to be almost identical to one for α equal to ∞. Thus, the assumption of a well mixed plug flow will be reasonably valid in this case for the values of α greater than approximately 3.

For a very slow and a very fast kinetics, the results shown in Fig. 6 indicate that the decrease in the value of α will degrade the performance of the reactor. On the other hand, for an intermediate range of the values of the kinetic constant, the decrease in the value of α found to give the higher values of f_A for the same system variables. The probable reason for this is that, as noted in the plug flow case, at very low or high values of the kinetic constants, wall permeation plays a very little role in improving the performance of the reactor. Thus under these conditions when the reaction process is kinetically controlled, the performance of a reactor would obviously be better for a well mixed plug flow than for a flow with large radial concentration gradients. The decrease in α, in general, increases the wall concentration build-up as shown for a typical condition in Fig. 5. Thus, the performance of a reactor would be degraded more and more with the decrease in the value of α at very high and low values of kinetic constants. In the intermediate range of kinetic constants, where the permeation has the most significant effect on the reactor performance, the decrease in the value of α found to improve the reactor performance as shown in Fig. 6. It should be noted though that from practical stand point the comparison of the values of f_A at various values of α and for the same values of K_1X and X is not quite correct because as shown in Fig. 5 the concentrations of the reactant at the tube wall are different for the different values of α. Hence, the achievement of the same permeation velocity at the different values of α but for otherwise identical conditions will, in general, require the different operating pressures. The higher operating pressure will be necessary at a lower value of α.

Suction with surface catalyzed reaction

At first thought it would appear that in the case of surface catalyzed reaction, a smaller value of α is always desirable because in this case the higher the concentration at the reactor wall the faster the reaction rate. The effect of the magnitude of α on the performance of a porous wall

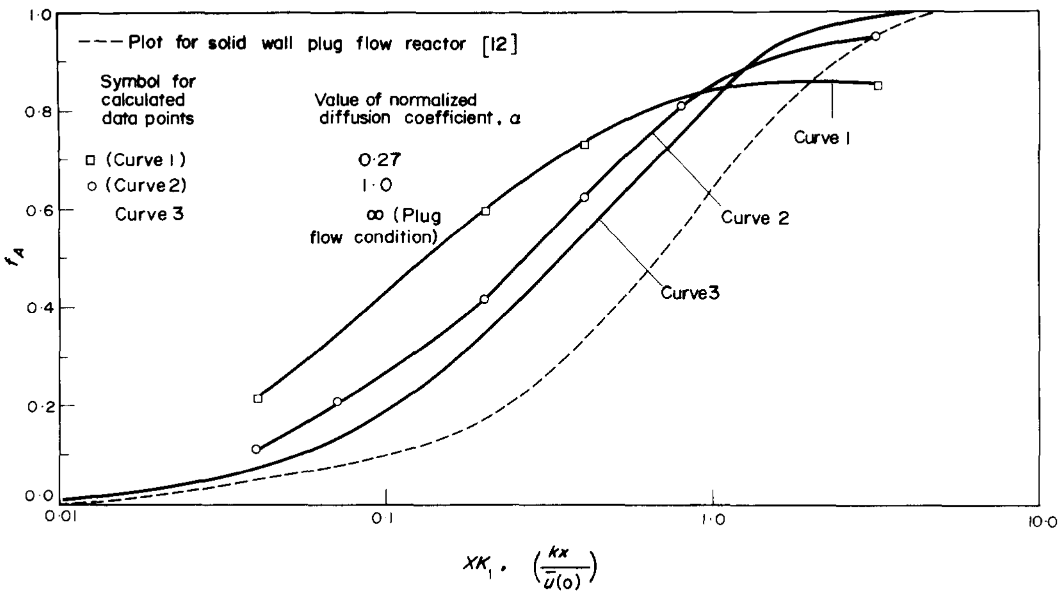

FIG. 7. Effect of normalized diffusion coefficient on the performance of a porous wall reactor with suction (X = 0·8, S = 1·0)—surface catalyzed reaction process in laminar flow condition.

reactor for a value of X equal to 0·8 is shown in Fig. 7. These results are obtained for the condition of no permeation of reactant through the wall. In this figure two values of α, 0·27 and 1·0, are examined. As expected, the value of f_A was increased significantly with the decrease in the value of α, for some values of K_1X. However, for the high values of K_1X the values of f_A found to be smaller for the smaller values of α. The curves for f_A vs. K_1X at various values of α

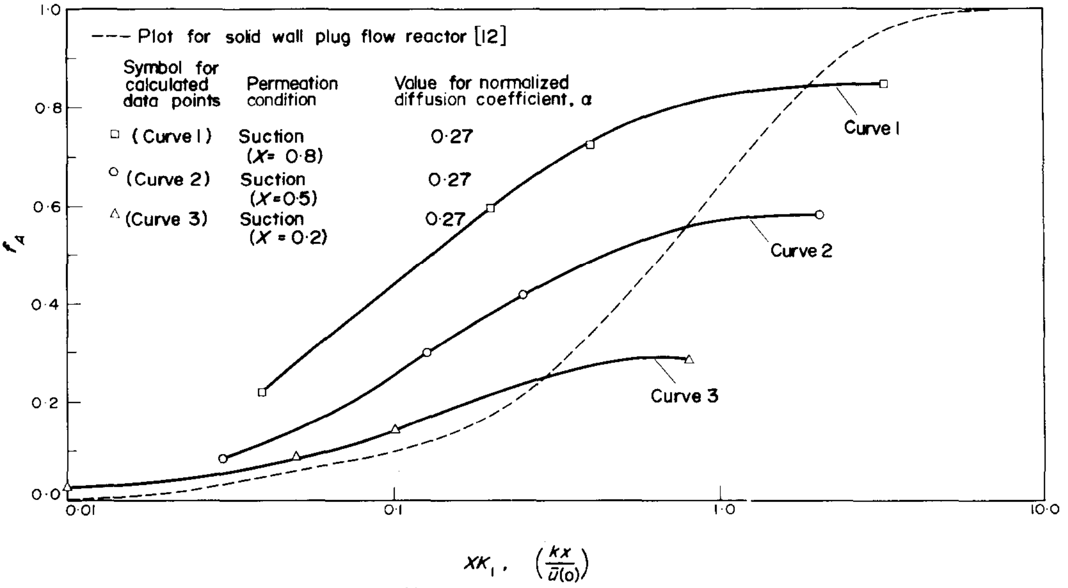

FIG. 8. Effect of quantity of permeation of inert and/or product on the performance of a porous wall reactor with suction (S = 1·0)—surface catalyzed reaction process in laminar flow condition.

follow the same trend as the one shown in Fig. 5 for the case of bulk phase reaction. An increase in the value of α would make a plot of f_A vs. K_1X more and more like the one for the case of well-mixed plug flow. For this case also the value of α greater than approximately 3 would mean the well-mixed plug flow condition within the reactor.

The effect of amount of wall permeation on the f_A vs. K_1X plot for a typical value of α equal to 0·27 is shown in Fig. 8. The results in this figure are obtained for the condition of no permeation of reactant through the tube wall. The results show the same behavior as the one obtained for the bulk phase reaction as shown in Fig. 4. The effect of X in Fig. 8, is however, more stronger than the one in Fig. 4. This is partly because the value of α for the results in Fig. 8 is smaller than that for the results shown in Fig. 4.

Effect of permeation of reactant through the wall

For both cases of bulk phase and surface catalyzed reactions, the permeation of reactant through the tube wall should always degrade the reactor performance. This is, of course, due to the fact that the reactant leaked through the wall is the lost amount which never takes part into the reaction. This effect of reactant leakage through the wall on the reactor performance for the typical reactor conditions is shown in Fig. 9. The results in this figure are obtained for the case of bulk phase reaction with the values of X equal to 0·5, α equal to 0·27 and the reactant rejection factor S equal to 0·8. The results of this figure validate the above line of reasoning.

Injection

The injection of reactant and/or inert in a porous wall reactor is more easily controllable process than that of suction. Hence in this case, the radial mass transfer can be reasonably well controlled.

The injection of the reactant will not normally create as prominent radial concentration profiles as obtained in the case of suction because the transverse velocity in this case will tend to create the radially uniform concentration. For this reason, in the present study the numerical

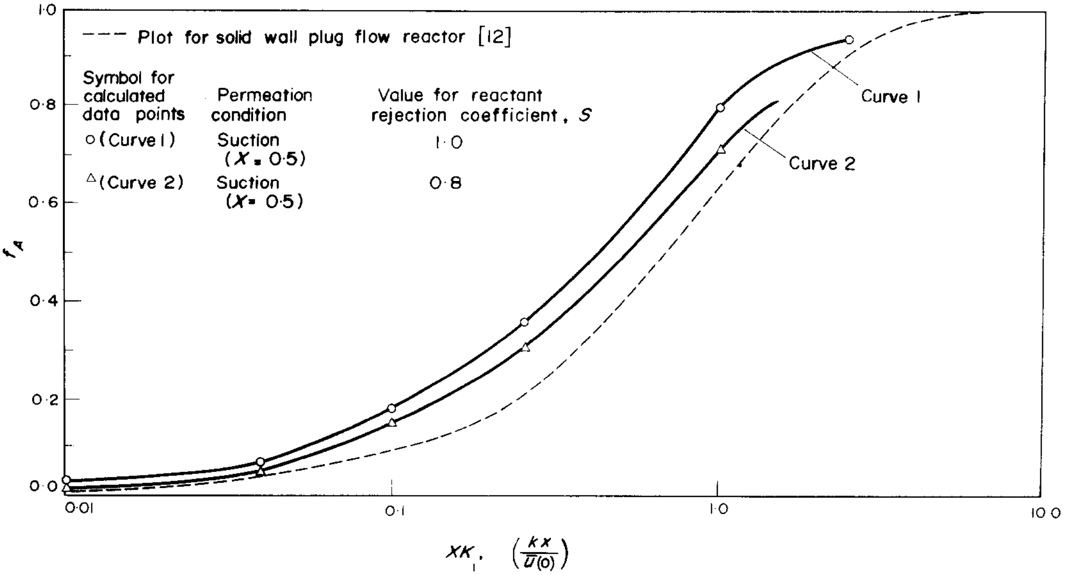

FIG. 9. Effect of transfer of reactant across the reactor wall on the performance of a porous wall reactor (α = 0·27)—bulk fluid reaction process in laminar flow condition.

results on the performance of a porous wall reactor in case of injection of reactant were not obtained.

The injection of pure inert in the system will always dilute the reacting mixture and thereby decrease the average of cup-mixing concentration. Thus injection of inert, as indicated in the plug flow analysis will, in general, degrade the performance of the reactor. This has been verified by Satterfield *et al.* [3] for the decomposition of hydrogen peroxide in a surface catalyzed porous wall reactor.

The numerical results for the reactor performance in case of inert injection in laminar flow condition were obtained. These results indicated that in this case concentration of the reactant at the center of the tube is higher than that near the wall. Also, the build-up of the reactant concentration at the tube center is higher for a smaller value of α. For bulk phase reaction process, unlike in the case of suction, the decrease in the value of α decreases the performance of a porous wall reactor for all the values of K_1X. However, just like in case of suction, higher the value of α, more closely the f_A vs. K_1X plot agrees with the one for a well-mixed plug flow condition for the same value of X.

In the case of surface catalyzed reaction process, the effect of α on the performance of the reactor was found to be opposite to one observed for the case of suction. This is to be expected because in this case smaller the value of α, smaller the concentration at the reactor wall. Just like in the case of suction, in this case also the values of α greater than approximately three gave the reactor performance close to one for the well-mixed plug flow. For both bulk phase as well as surface catalyzed reaction processes, an increase in amount of injection of inert found to have an opposite effect on the reactor performance to the one obtained for the increase in amount of suction of inert. Since all the results for the case of injection of inert which are described so far are the logical extensions of ones described for the case of suction of inert or product, they are not graphically displayed here.

Practical utility of the analysis

The theoretical study presented here is a part of a continuing study on the applications of semi-permeable polymeric membranes or other types of porous materials to the field of chemical reaction engineering. As it may be obvious, the practical implementation of the concept of a porous wall reactor depends upon the availability of the suitable porous materials. If the reactor is to be operated with the injection of reactants or inert materials, a suitable porous material should be easy to find, and with this wall condition, the analysis presented here should be directly applicable to the reactions such as hydrolysis of acetic anhydride [11] and the decomposition of hydrogen peroxide [3]. The former of these two reactions is a bulk phase, homogeneous reaction; while the latter one is a heterogeneous, surface catalyzed reaction. When the permeation condition is to be that of suction, a suitable reactor material will be, at present, difficult to find, and most probably a semi-permeable membrane will have to be used for this purpose. As reported by Wang and Humphrey [14] and Wang *et al.* [15], there are presently some membranes available which can be used for this purpose for some low temperature, enzymatic reactions, such as conversion of starch into glucose etc. The analysis presented here can be applicable to these enzymatic reactions when they can be approximated by a pseudo first order reaction mechanism.

In the absence of accurate experimental data on the permeation characteristics of the suitable materials required for any of the reaction systems mentioned above, the practical utility of the present analysis is illustrated here with a set of assumed data. The sample calculations are carried out for both liquid as well as gas phase reactions. Thus, it is assumed,
for liquid phase reaction:

$$D_A = 5{\cdot}0 \times 10^{-5}\ \mathrm{ft}^2/\mathrm{h}$$
$$\nu = 0{\cdot}036\ \mathrm{ft}^2/\mathrm{h}$$

$$k = 0{\cdot}1 \ \text{l/h}$$

for gas phase reaction:

$$D_A = 0{\cdot}30 \ \text{ft}^2/\text{h}$$

$$\nu = 0{\cdot}36 \ \text{ft}^2/\text{h}$$

$$k = 0{\cdot}1 \ \text{l/h}.$$

The above listed data for D_A and ν are the typical values for each phase obtained from [16] and [17]. The reactor diameter is chosen to be 0·2 ft and the average tube inlet velocity of the reacting mixture is set at 10 ft/h. It is now desired to know the performance of a reactor with given wall characteristics as a function of reactor length.

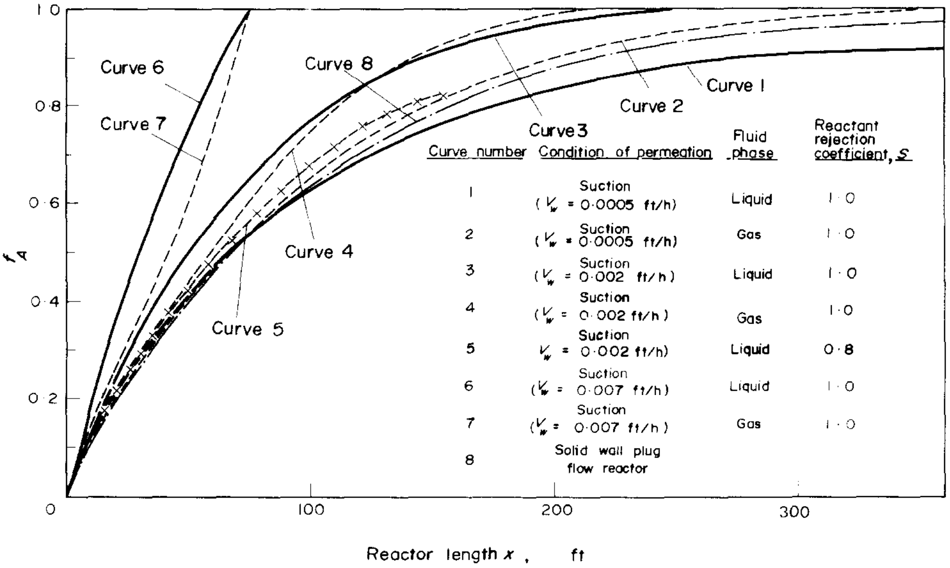

FIG. 10. Design curves for the sample example.

Figure 10 shows the plots of f_A vs. reactor length x for the data used in this sample example. These plots are obtained using the results such as ones shown in Figs. 4, 6 and 9. The solid curves in Fig. 10 are the plots of f_A vs. x for the liquid phase reaction at the various values of suction velocity. The dashed curves are the similar plots for the gas phase reaction. All of these plots are calculated assuming that there is no transfer of reactant across the tube wall. Thus, they represent the best performances of a porous wall reactor with suction, for the various values of suction velocity. For one value of suction velocity (v_w = 0·002 ft/h) and for the liquid phase reaction, a plot of f_A vs. x was also calculated for the value of S equal to 0·8. This plot is shown as a crossed curve in Fig. 10. As noted above, this curve lies below the similar curve for the value of S equals to 1·0. In order to illustrate the relative performances of a porous wall reactor with that of a solid wall reactor, Fig. 10 also shows a plot of f_A vs. x for latter type of the reactor. It should be noted that the plots very similar to the ones shown in Fig. 10 can be obtained if the reaction is surface catalyzed instead of homogeneous.

Finally, the practical applications of the present analysis also demand some comments on the performance of a porous wall reactor when the reaction and the permeation conditions are other than the ones assumed here. If the reaction order is other than one, or if the reaction is reversible, or if the permeation velocity is nonuniform and a function of axial distance, the analytical technique used here will not be directly applicable. Under these conditions, a suitable numerical technique such as one illustrated by Brian [7] will have to be used. When the injection or suction is non-uniform but the reaction is first order, the performance of the porous wall reactor may, however, be approximately evaluated with the present analysis by using an average (suitably arithmetic) value of the permeation velocities at the reactor inlet and outlet in the calculations of X and α.

CONCLUSIONS

For the well-mixed plug flow condition, a porous wall characteristics will improve the performance of a reactor best when the wall completely rejects reactant in the case of suction and completely rejects inert and product in the case of injection. These results are valid for the bulk phase as well as the surface catalyzed reaction processes. In case of laminar flow, the radial mass transfer will have a significant effect on the reactor performance for the value of normalized diffusion coefficient less than approximately 3·0. This effect is significant for both bulk phase as well as surface catalyzed reaction processes and it would be more important for the cases of suction and the injection of inert or product than it would be for the cases of suction and injection of the reactant. Since the value of α equal to 3 is higher than ones commonly encountered in liquid systems, the radial diffusive mass transfer will play very significant role in determining the performance of a porous wall reactor for a liquid phase reaction. However, for a gaseous reacting system, unless the permeation velocity is very high, the radial diffusive mass transfer will not be very important. Finally, this paper presents an efficient method of representation of the performance of a porous wall reactor under a wide range of reaction, flow and mass transfer conditions.

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EFFETS DU TRANSFERT MASSIQUE RADIAL DANS UN RÉACTEUR TUBULAIRE À PAROI POREUSE

Résumé—Le réacteur tubulaire à paroi poreuse avec une suction ou une injection sélective de réactant et (ou) de produit et (ou) de corps inerte offre de nombreux avantages sur le réacteur tubulaire conventionnel à paroi solide. Mais l'usage de ce type de réacteur demande une connaissance complète des effets du transfert massique radial sur les performances du réacteur. L'article analyse ces effets dans des conditions d'écoulement piston aussi bien que laminaire et pour un processus de réaction isotherme et irréversible du premier ordre. On considère à la fois la phase globale et la réaction catalysée en surface. Les résultats de l'analyse sont présentés et interprétés en fonction de la quantité de réactant entrant et consommé par la réaction dans le réacteur pour une variété de réactions, d'écoulements et de conditions de transfert massique radial.

EFFEKTE DES RADIALEN MASSENAUSTAUSCHES IN EINEM ROHRREAKTOR MIT PORÖSER WAND

Zusammenfassung—Ein Rohrreaktor mit poröser Wand für selektive Absaugung oder Zuführung der Ausgangsstoffe und/oder der Produkte und/oder von Inertstoffen bietet viele Vorteile gegenüber einem konventionellen Rohrreaktor mit fester Wand. Aber die Handhabung dieses Reaktortyps erfordert eine gründliche Kenntnis der Effekte des radialen Massenaustauschs auf die Arbeitsweise des Reaktors. Die vorliegende Veröffentlichung analysiert diese Effekte für den Fall der Pfropfenströmung und der laminaren Strömung und für eine isotherme irreversible Reaktion erster Ordnung. Es werden Prozesse mit homogener Reaktion und mit Wandkatalyse betrachtet. Die Ergebnisse der Untersuchung werden dargestellt in Abhängigkeit von der Menge des zugeführten Ausgangsstoffes, die durch die Reaktion im Reaktor abnimmt, bei Variation der Reaktions-, Strömungs- und Massenaustausch-Bedingungen.

ВЛИЯНИЕ РАДИАЛЬНОГО МАССООБМЕНА В ТРУБЧАТОМ РЕАКТОРЕ С ПОРИСТЫМИ СТЕНКАМИ

Аннотация—По сравнению с обычным трубчатым реактором с твёрдыми стенками при избирательном отсосе или вдуве реагирующего или нереагирующего вещества или продукта реакция обладает большими преимуществами. Однако, использование этого типа реактора требует тщательных знаний о влияниях радиального массообмена на работу реактора. В данной статье анализируются влияния в условиях стержневого, а также ламинарного течений для изотермического необратимого процесса первого порядка. Рассматриваются реакции в газовом объёме и на каталитической поверхности, Приводятся и обсуждаются результаты анализа при различных реакциях и условиях течения и радиального массообмена.